Precalculus Review Chapter 7

I. Simplify each expression

1. $\dfrac{\sec x}{\tan x}$

$$\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \csc x$$

2. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$$\sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$$

$$(\sin^2 x + \cos^2 x) + (\sin^2 x + \cos^2 x)$$

$$1+1$$

$$2$$

II. Verify that each equation is an identity

3. $\tan x = \dfrac{\sec x}{\csc x}$

$\tan x$

$$\frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$$

4. $\sec x \csc x = \tan x + \cot x$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} \qquad \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} \qquad \frac{1}{\sin x \cos x}$$

5. $\dfrac{1+\tan x}{\sin x + \cos x} = \sec x$

$$\frac{1 + \frac{\sin x}{\cos x}}{\sin x + \cos x}$$

$$\frac{\frac{\cos x + \sin x}{\cos x}}{\sin x + \cos x} = \frac{\sin x + \cos x}{\cos x} \cdot \frac{1}{\sin x + \cos x} = \frac{1}{\cos x} = \sec x$$

6. $\csc x - 1 = \dfrac{\cot^2 x}{\csc x + 1}$

$$\frac{\csc^2 x - 1}{\csc x + 1} = \frac{(\csc x + 1)(\csc x - 1)}{(\csc x + 1)}$$

$$\csc x - 1$$

III. Use the sum or difference identities to find the exact value of each trigonometric function.

7. $\sin 165°$

$$\sin(120° + 45°) = \sin 120° \cos 45° + \cos 120° \sin 45°$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

8. $\tan \dfrac{23\pi}{12}$

$$\tan\left(\frac{5\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{5\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$$

$$\frac{\sqrt{3}+1}{1-(\sqrt{3})(1)} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

IV. Find the exact value of x and y.

9. $\sin(x+y)\, if\ \sin x = \frac{9}{41}\ and\ \cos y = \frac{3}{5}$

$\cos^2 x = 1-\left(\frac{9}{41}\right)^2 = \frac{1600}{1681}$

$\sin^2 y = 1-\left(\frac{3}{5}\right)^2 = \frac{16}{25}$

$\cos x = \frac{40}{41}$

$\sin y = \frac{4}{5}$

$$\sin(x+y) = \sin x\cos y + \cos x \sin y$$

$$= \left(\frac{9}{41}\right)\left(\frac{3}{5}\right)+\left(\frac{40}{41}\right)\left(\frac{4}{5}\right) = \frac{187}{205}$$

IV. Use a half-angle identity to find the exact value of each function.

10. $\sin 15°$

$$\sin\frac{30}{2} = \pm\sqrt{\frac{1-\cos 30}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2}$$

Answer is positive because 15⁰ is in quadrant I.

11. $\tan\frac{5\pi}{12}$

$$\tan\frac{\frac{5\pi}{6}}{2} = \pm\sqrt{\frac{1-\cos\frac{5\pi}{6}}{1+\cos\frac{5\pi}{6}}} = \pm\sqrt{\frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{1+\left(\frac{-\sqrt{3}}{2}\right)}}$$

Answer is positive because 5π/12 is in quadrant I.

$$= \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$$

V. Use the given information to find the following.

$$\sin 2\theta, \cos 2\theta, \tan 2\theta$$

12. $\cos\theta = \frac{2}{3}, 0 < \theta < 90°$

$\sin^2\theta = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$

$\sin\theta = \frac{\sqrt{5}}{3}$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right) = \frac{4\sqrt{5}}{9}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{2}{3}\right)^2 - 1 = \frac{-1}{9}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4\sqrt{5}}{9}}{\frac{-1}{9}} = \frac{4\sqrt{5}}{9}\cdot\frac{9}{-1} = -4\sqrt{5}$$

VI. Verify that each equation is an identity.

13. $\csc 2\theta = \frac{1}{2}\sec\theta\csc\theta$

| | |
|---|---|
| $\frac{1}{\sin 2\theta}$ | $\frac{1}{2}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$ |
| $\frac{1}{2\sin\theta\cos\theta}$ | $\frac{1}{2\sin\theta\cos\theta}$ |

VII. Solve each equation for x between $0^0$ and $360^0$.

14. $\cos x \tan x = \frac{1}{2}$

$\cos x\left(\frac{\sin x}{\cos x}\right) = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = 30°, 150°$

15. $\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$ *or* $\tan x - 1 = 0$

$\tan x = 1$

$x = 0°, 180°$ $x = 45°, 225°$

16. $2\cos^2 x = 3\sin x$

$2(1 - \sin^2 x) = 3\sin x$

$2 - 2\sin^2 x = 3\sin x$

$-2\sin^2 x - 3\sin x + 2 = 0$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$2\sin x - 1 = 0$ *or* $\sin x + 2 = 0$

$2\sin x = 1$ $\sin x = -2$

This solution does not work

$\sin x = \frac{1}{2}$

$x = 30°, 150°$

VIII. Write the standard form of the equation from the given information.

17. $p = 12, \phi = 45°$

$x\cos 45° + y\sin 45° - 12 = 0$

$x\left(\frac{\sqrt{2}}{2}\right) + y\left(\frac{\sqrt{2}}{2}\right) - 12 = 0$

$x\sqrt{2} + y\sqrt{2} - 24 = 0$

$x\sqrt{2} + y\sqrt{2} = 24$

IX. Write the equation in normal form. Find the length of the normal and the angle it makes with the positive x-axis.

18. $\frac{x}{2} - 3y = 1$

$x - 6y = 2$

$x - 6y - 2 = 0$

$\sqrt{A^2 + B^2} = \sqrt{1^2 + (-6)^2} = \sqrt{37}$

$\frac{1}{\sqrt{37}}x - \frac{6}{\sqrt{37}}y - \frac{2}{\sqrt{37}} = 0$

$\cos\phi = \frac{\sqrt{37}}{37}$

$\frac{\sqrt{37}}{37}x - \frac{6\sqrt{37}}{37} - \frac{2\sqrt{37}}{37} = 0$

$\phi = \cos^{-1}\frac{\sqrt{37}}{37}$

$\phi = 80.5°$

$p = \frac{2\sqrt{37}}{37}$

X. Find the distance between the given point and the given line.

19. $(2,4), 2x + 3y - 4 = 0$

$d = \frac{2(2) + 3(4) - 4}{\sqrt{2^2 + 3^2}}$

$d = \frac{4 + 12 - 4}{\sqrt{4 + 9}}$

$d = \frac{12}{\sqrt{13}}$

$d = \frac{12\sqrt{13}}{13}$

I went to Mr. Frey's Precalculus website and all I got was this lousy 5 point bonus coupon.

Signed________________________________